Trig Functions + Uncertainties

No simple rule .... the uncertainty is half of the difference between the highest and lowest values.

$$\cos\theta = ? \quad \text{if} \quad \theta = (60 \pm 5)^{\circ}$$

$$\cos 60^{\circ} = 0.500$$

$$\cos 65^{\circ} = 0.423$$

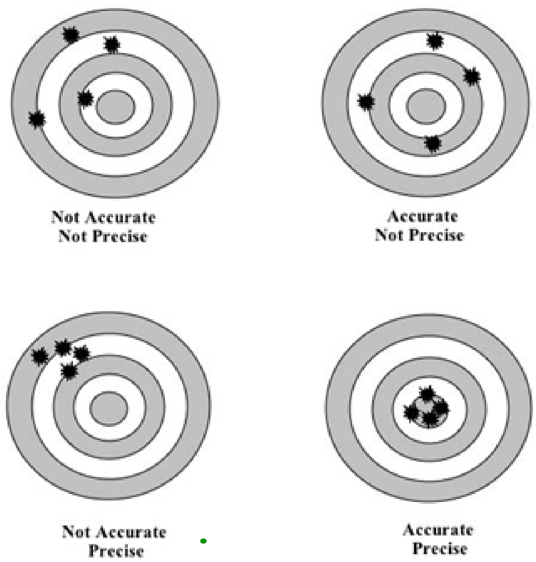
$$\cos 55^{\circ} = 0.574$$

$$\frac{0.574 - 0.423}{2} = 0.076$$

$$\cos(60 \pm 5)^{\circ} = 0.50 \pm 0.08$$

NOTE: (in general)

If one uncertainty is very large large compared to another, then you may choose to only work with the very large uncertainty



(52 ± 1) cm

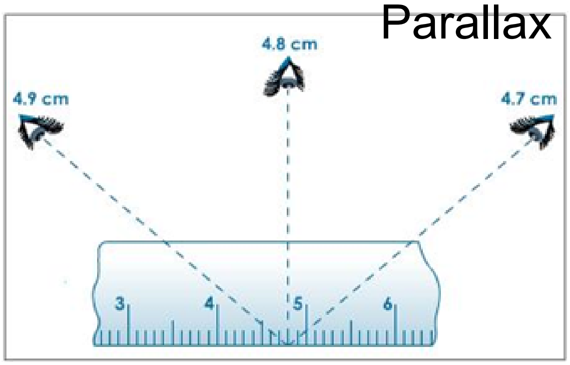
$(52.\cancel{1 \pm} 1)$ cm

LC = 1mm

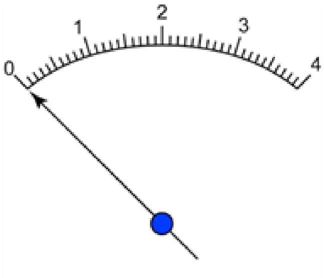
if ball is moving ±1cm

(Image Source: http://celebrating200years.noaa.gov/magazine/tct/accuracy_vs_precision.html

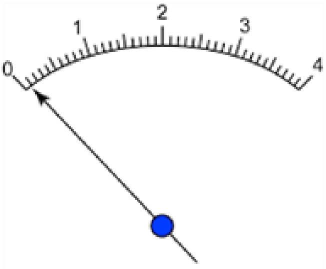
Parallax



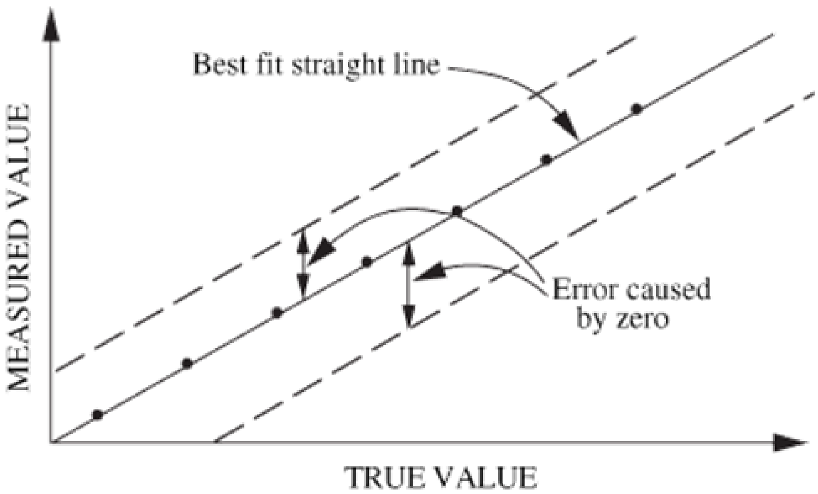
(Image Source: http://www.tutorvista.com/content/physics/physics-i/measurement-and-experimentation/measurement-length.php)



No zero error

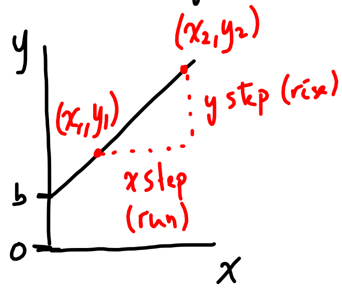


Positive zero error



Interpretation of Linear Graphs

Consider a linear graph (where $b \neq 0$)



$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

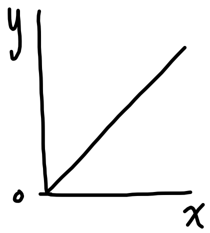
$$\text{slope} = \frac{y\ \text{step}}{x\ \text{step}}$$

$$\text{slope} = \frac{\Delta y}{\Delta x}$$

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

Recall: $y = mx + b$

What if you have a linear relationship with $b = 0$?



This is called a direct proportionality between y and x

"y is directly proportional to x"

"y varies directly with x"

a proportionality statement → $y \propto x$ $\propto$ $\propto$

"is proportional to"

If you double x then y is doubled as well

$y \propto x$ (proportionality statement)

$y = kx$ (general equation)

k is the proportionality constant.

$(y = mx + b)$

A graph of y vs x will be linear with a slope of k and a y-intercept of zero!

Not all data you plot will give you a linear graph!

<u>Power Curve</u>

$y \propto x^n$

$y = k x^n + 0$

$(y = m x + b)$

a linear equation "in disguise"

A graph of y vs x^n will be linear with a slope of k and a y-interupt of zero.

linear graph (b=0) $\Rightarrow y \propto x^n$

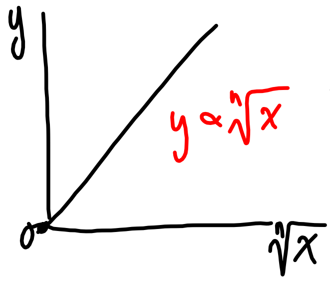
(y is proportional to x^n)

Root Curve

$y \propto \sqrt[n]{x}$

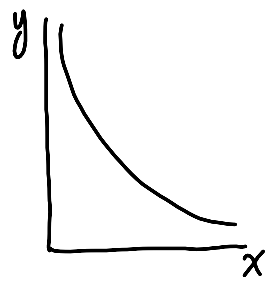
$y = k\sqrt[n]{x}$

$(y = mx + b)$



A graph of y vs $\sqrt[n]{x}$ will be linear with a slope of k and a y-intercept of zero)

Inverse Curve



$y \propto \frac{1}{x^n}$

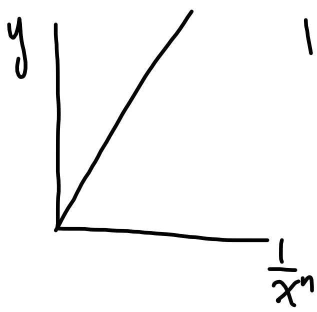
"y is inversely proportional to x^n"

$y = k\left(\frac{1}{x^n}\right)$

"y is directly proportional to $\frac{1}{x^n}$"

$(y = mx + b)$

A graph of y vs $\frac{1}{x^n}$ will be linear with a slope of k and a y-int of zero.



Summary:

Power: $y \propto x^n$

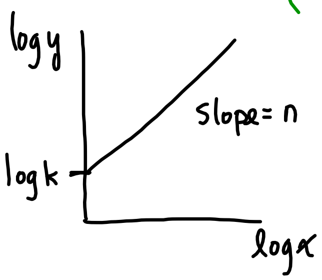
Root: $y \propto \sqrt[n]{x}$ $\quad (y \propto x^{\frac{1}{n}})$

Inverse: $y \propto \frac{1}{x^n}$ $\quad (y \propto x^{-n})$

all three can be expressed with n in the exponent.

Consider that you need to find the value for n:

$$y \propto x^n$$
$$y = kx^n$$
$$\log y = \log(kx^n)$$
$$\log y = \log k + \log x^n$$
$$\log y = \log k + n \log x$$
$$(y = b + mx)$$



A graph of log y vs log x will be linear with a slope of n and a y-intercept of log k

$b = \log k$

$k = 10^b$

(inverse log)

Consider $T = 2\pi\sqrt{\frac{l}{g}}$ where g is a constant.

$$T = \frac{2\pi}{1}\frac{\sqrt{l}}{\sqrt{g}}$$

$T \propto \sqrt{l}$ → $$T = \frac{2\pi}{\sqrt{g}}\frac{\sqrt{l}}{1}$$

$$y = mx + b$$

A graph of T vs $\sqrt{l}$ will be linear with a slope of $\frac{2\pi}{\sqrt{g}}$ and a y-intercept of zero.